

Holographic screens in ultraviolet self-complete quantum gravity

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Abstract

In this paper we study the geometry and the thermodynamics of a *holographic screen* in the framework of the ultraviolet self-complete quantum gravity. To achieve this goal we construct a new static, neutral, non-rotating black hole metric, whose outer (event) horizon coincides with the surface of the screen. The space-time admits an extremal configuration corresponding to the minimal holographic screen and having both mass and radius equaling the Planck units. We identify this object as the space-time fundamental building block, whose interior is physically inaccessible and cannot be probed even during the Hawking evaporation terminal phase. In agreement with the holographic principle, relevant processes take place on the screen surface. The area quantization leads to a discrete mass spectrum. An analysis of the entropy shows that the minimal holographic screen can store only one byte of information while in the thermodynamic limit the area law is corrected by a logarithmic term.

1 Introduction

“Quantum gravity” is the common tag for any attempt to reconcile gravity and quantum mechanics. Since the early proposals by Wheeler and deWitt, up to the recent ultraviolet (UV) self-complete scenario, the diverse formulations of a would be quantum theory of gravity have shown a common feature, i.e. a

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fundamental length/energy scale where the smooth manifold model of space-time breaks down. Let us refer to this scale as the “Planck scale” irrespectively whether it is 10^{19} GeV or, $10 - 10^2$ TeV. The very concept of distance becomes physically meaningless at the Planck scale and spacetime “evaporates” into something different, a sort of “foamy” structure, a spin network, a fractal dust, etc., according with the chosen model.

As a matter of fact, one of the most powerful frameworks for describing the Planckian phase of gravity is definitely (Super)String Theory. The price to pay to have a perturbatively finite, anomaly-free quantum theory is to give up the very idea of point-like building blocks of matter, and replace them with one-dimensional vibrating strings. As there does not exist any physical object smaller than a string, there is no physical ways to probe distances smaller than the length of the string itself.

In this regard two properties of fundamental strings are worth mentioning:

- string excitations corresponds to different mass and spin “particle” states;
- highly excited strings share various physical properties with black holes.

Thus, we infer that string theory provides a bridge between particle-like objects and black holes (see for instance [1]).

However, it is important to remark that while the Compton wavelength of a particle-type excitation decreases by increasing the mass, the Schwarzschild radius of a black hole increases with its mass. Thus, the first tenet of high energy particle physics, which is “higher the energy shorter the distance”, breaks down when gravity comes into play and turns a “particle” into a black hole.

The above remark is the foundation of the UV self-complete quantum gravity scenario, where the Planckian regime is permanently shielded from observation by the production of black hole excitations in Planck energy scattering [2]. From this “reverse perspective”, trans-Planckian physics is dominated by larger and larger black hole configurations. It follows that only black hole larger, or at most equal to Planck size objects, can self-consistently fit into this scheme. A family of black objects endowed with such a property have been found on the ground of noncommutative geometry arguments and studied for different physical conditions in a series of papers [3] (for an “incomplete” list of the latest developments in such a quickly growing literature see also [4,5,6,7,8,9,10] while for a review see [11]).

A characteristic feature of this type of black holes is that the minimum size configuration is “extremal” even in the neutral non-spinning case. The stabilization effect is obtained by introducing, from the very beginning, a fundamental length in the metric, determining the size of the extremal configuration. The fundamental length being a phenomenological input from quantum gravity, whatever it is. These objects fit pretty well in the UV self-complete scenario providing a stable, minimum size, probe at the transition point between particles and black holes [12].

In this paper we want to take a step further in the realization of this program

by avoiding the introduction of a unspecified minimal length and replacing it with the radius of a *minimal holographic screen* representing the building block spacetime is supposed to be made of. Our strategy is to merge ideas from UV self-complete quantum gravity and holography to produce a self-consistent scheme where gravity provides by itself its own short distance regulator effectively cutting off trans-Planckian physics.

The paper is organized as follows. In Section (2) we “engineer” a new black hole metric modeling spacetime outside an holographic screen by properly defining the total mass energy. We show that the surface of the screen coincides with the outer horizon of the black hole and that the mass spectrum is bounded from below by the Planck mass. In Section (3) we discuss the thermodynamics of the screen. We find that the area law is modified by logarithmic corrections and the minimal holographic screen has zero entropy. Finally we propose an “holographic” quantization scheme where the area of the extremal configuration provides the “quantum” of surface. In Section (4) we exploit the results of the previous section to address the issue of Newton’s law as an entropic force in UV self-complete quantum gravity scenario. In Section (5) we offer to the reader a brief summary of the main results of this work. In the Appendix we build the energy momentum tensor sourcing the holographic screen metric in the framework of general relativity.

2 Self-regular black hole solution

In reference [13] a simple but intriguing model of singularity-free black hole was “guessed”, in the sense that the metric was assigned as an in-put for the Einstein equations and the source was “derived” from them. Sometime this inverted procedure is called “engineering” a solution of the field equations and is frequently used in the framework of wormholes modeling. The distinctive feature of the solution is the presence in the line element of a free parameter with dimension of a length, acting as a short distance regulator for the spacetime curvature, allowing a safe investigation of back-reaction effects of the Hawking radiation.

The solution proposed in [13] has been recently extended to higher dimensions by one of the authors in [14], where it was also shown that by a numerical rescaling of the short-distance regulator it was possible to identify this fundamental length scale with the radius of the extremal configuration.

Along the lines of the above contributions the natural step forward would be to have a black hole that enjoys the following properties

- i) no curvature singularity in the origin;
- ii) Schwarzschild behavior for distances bigger than a characteristic minimal length scale l_0 ;

- iii) self-encoding the characteristic scale l_0 in the spacetime geometry by means of the radius of the extremal configuration r_0 , i.e., $r_0 = l_0$.

The latter condition is crucial. For instance noncommutative geometry inspired black holes [11] already enjoy both i) and ii), but fail to fulfill the condition iii). This means that the characteristic length scale of the system l_0 and the extremal configuration radius r_0 are independent quantities. Indeed noncommutative geometry is the underlying theory which provides the scale l_0 in terms of an “external” parameter, namely the noncommutative parameter θ . In other words one needs to invoke a principle, like a modification of commutators in quantum mechanics, or the emergence of a quantum gravity induced fundamental length to achieve the regularity of the geometry at short scales. Against this background, we want just to use r_0 as fundamental scale, getting rid of any l_0 as emerging from any theory or principle not included in Einstein field equations. This is a step forward since it opens the possibility for Einstein gravity to be self-protected in the ultraviolet regime. We start from

$$ds^2 = - \left(1 - \frac{2MG}{r} h_\alpha(r) \right) dt^2 + \left(1 - \frac{2MG}{r} h_\alpha(r) \right)^{-1} dr^2 + r^2 d\Omega^2 \quad (1)$$

where $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$ and $h_\alpha(r)$, a mass profile function, is

$$h_\alpha(r) = \frac{r^3}{[r^\alpha + (\tilde{r}_0)^\alpha / 2]^{3/\alpha}} \quad (2)$$

with \tilde{r}_0 and α are free parameters. Since we want to work in the framework of Einstein gravity, there exists only one additional scale beyond r_0 , i.e. the Planck length $L_P = \sqrt{G}$, or the Planck mass $M_P = 1/\sqrt{G}$. *By requiring r_0 to be the only universal scale in the system we must select $h_\alpha(r)$ in order to have $r_0 = L_P$ and the extremal black hole mass $M_0 = M_P$.* It is not hard to prove that the conditions i) and ii) are met for any $\alpha > 0$. In addition, we see that for any $\alpha > 0$ the radius of the extremal configuration is exactly $r_0 = \tilde{r}_0$. Therefore we can drop the tilde from the expression above and set $r_0 = L_P$. In order to fulfill the additional condition $M_0 = M_P$ we need to select a specific value of α . From $M_0 = M(r_0)$ we find

$$M_0 = \frac{1}{2} \left(\frac{3}{2} \right)^{3/\alpha} \left(\frac{r_0}{L_P} \right) M_P. \quad (3)$$

Equation (3) shows that the extremal black hole configuration has a minimum mass, given by the Planck mass, provided we choose the value

$$\alpha = \alpha_0 \equiv \frac{3}{\ln 2} \times \ln \frac{3}{2} \simeq 1.75 \longrightarrow M_0 = M_P, \quad r_0 = L_P. \quad (4)$$

By a proper choice of the α parameter, we determined a black hole mass spectrum which bounded from below by the Planck mass. The lightest, and

smallest, black hole in this family is a Planck size, extremal, object. Particle-like objects with $M < M_P$ can never collapse into a black hole but give rise to regular, horizon-less metrics.

By restoring the gravitational constant $G = L_P^2$ we can display the metric as

$$ds^2 = - \left(1 - \frac{2ML_P^2 r^2}{(r^{\alpha_0} + L_P^{\alpha_0}/2)^{3/\alpha_0}} \right) dt^2 + \frac{dr^2}{\left(1 - \frac{2ML_P^2 r^2}{(r^{\alpha_0} + L_P^{\alpha_0}/2)^{3/\alpha_0}} \right)} + r^2 d\Omega^2. \quad (5)$$

Despite the virtues of the above metric, we feel that the set of conditions i), ii) and iii) over-specify the problem and a further step forward is possible. Having in mind that for extremal black hole configurations the Hawking emission stops we just need to find a metric for which only the conditions ii) and iii) hold. These would be enough for completing the program of the UV self-complete quantum gravity by protecting the short distance behavior of gravity during the final stages of the evaporation process. In this regards, the resulting extremal black hole is just the smallest object one can use to probe short-distance physics, irrespective of the behavior of the metric behind the degenerate horizon. In other words, in the framework of UV self-complete quantum gravity, *it is not physically meaningful to ask about curvature singularity inside the horizon as the very concept of spacetime is no longer defined below this length scale*. According with such a line of reasoning, we can determine a new profile for the function $h(r)$ by dropping the condition i). As a result we find the following metric (for a derivation of the metric as a classical solution of the Einstein equation, please see the Appendix)

$$ds^2 = - \left(1 - \frac{2ML_P^2 r}{r^2 + L_P^2} \right) dt^2 + \left(1 - \frac{2ML_P^2 r}{r^2 + L_P^2} \right)^{-1} dr^2 + r^2 d\Omega^2. \quad (6)$$

where the arbitrary constant M is defined as follows:

$$M \equiv \frac{1}{2L_P^2 r_h} (r_h^2 + L_P^2) \quad (7)$$

We give M the physical meaning of mass for a radius r_h , spherical, *holographic screen*. Several remarks are in order.

- It is easy to show that $M \geq M_P$ and equals the Planck mass only for $r_h = L_P$.
- The line element (6) admits a pair of horizons provided $M \geq M_P$. The radii r_{\pm} of the horizons are given by

$$r_{\pm} = L_P^2 \left(M \pm \sqrt{M^2 - M_P^2} \right) \quad (8)$$

For $M = M_P$ the two horizons merge into a single (degenerate) null surface at $r_{\pm} = r_0 = L_P$. For $M \gg M_P$ the outer horizon approaches the conventional value of the Schwarzschild geometry, i.e., $r_+ \simeq 2ML_P^2$.

- By inserting (7) into (8) one finds

$$r_+ = r_h , \quad (9)$$

$$r_- = \frac{L_P^2}{r_h} \quad (10)$$

We see that the holographic screen surface coincide with the (outer) black hole horizon r_+ , while the inner Cauchy horizon has a radius which is always smaller or equal to the Planck length. This fact lets us circumvent the issue of potential blue shift instabilities (see for instance [15] for recent analyses of quantum gravity corrected metrics) because r_- simply loses its physical meaning being not accessible to any sort of measurement process.

In what follows we can identify the holographic screen with the black horizon without distinguishing anymore between the two surfaces.

- “Light” objects, with $M < M_P$, are “particles” rather than holographic screens. By particles we mean localized lumps of energy of linear size given by the Compton wavelength $\lambda = 1/M$. Thus, they cannot probe distances smaller than λ . The “transition” particle \rightarrow black holes is discussed below in terms of critical *surface density*.

We conclude that the metric (6) provides for $M = M_P$ a model of *minimal holographic screen*, i.e., the “atom” or “building block” the spacetime is supposed to be made of. We reached the ultimate frontier, where space and time dissolve into their fundamental constituents.

It is interesting to consider the *surface energy density* of the holographic screen which is defined as

$$\sigma_h \equiv \frac{M}{4\pi r_h^2} = \frac{1}{8\pi L_P^2} \frac{r_+^2 + L_P^2}{r_+^3} . \quad (11)$$

From the above relation we see that σ_h is a monotonically decreasing function of the screen radius (see also Fig. 1 for σ_h as a function of M). We notice that there exists a minimal screen encoding the physically maximum attainable energy density, i.e. the Planck (surface) density:

$$\sigma_h (r_+ = L_P) = \frac{1}{4\pi L_P^3} = \frac{M_P}{4\pi L_P^2} \quad (12)$$

We stress that there is no physically meaningful “interior” for the minimal screen, i.e. the “volume” of such an object is not even defined. Thus, we can only consider energy per unit area, rather than per unit volume.

If we, formally, define a surface energy for a particle as

$$\sigma_p \equiv \frac{M}{4\pi \lambda^2} = \frac{1}{4\pi \lambda^3} \quad (13)$$

we see that the two curves (11) and (11) cross at $\lambda = L_P = r_+$. Thus, the Planck density (12) is the *critical density* for a particle to collapse into a black hole. This argument is usually formulated in terms of volume energy density

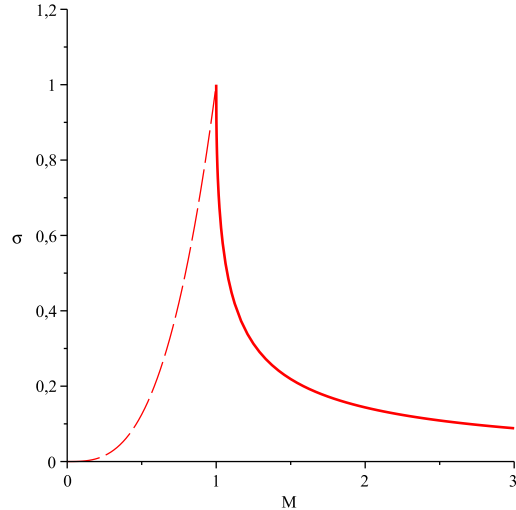


Fig. 1. The normalized surface energy density as a function of the mass M in Planck units. The dashed curve represents the surface density of the particle σ_p while the solid curve is the holographic screen surface density σ_h .

having in mind the picture of macroscopic body gravitationally collapsing under their own weight. From our holographic vantage point, where “surfaces” are the basic dynamical objects, it is natural to reformulate this reasoning in terms of areal densities.

In addition holography offers a way to circumvent potential conflicts between the mechanism of spontaneous dimensional reduction [16] and the UV self complete paradigm. If we perform the limit for $r \rightarrow 0$ the metric (6) would apparently reduce into an effective two-dimensional spacetime

$$ds^2 \longrightarrow -(1 - 2Mr) dt^2 + (1 - 2Mr)^{-1} dr^2 + \mathcal{O}(r^2/L_P^2). \quad (14)$$

As explained in [17], this mechanism would lead the formation of lower dimensional black holes for length scales below the Planck length, in contrast with the predicted semi-classical regime of trans-Planckian black holes in four dimensions. However, contrary to the Schwarzschild metric that eventually reduces into dilaton gravity black holes when $r \simeq L_P$ (for reviews of the mechanism see [18]), the presence of the holographic screen forbids the access to length scales $r < L_P$ and safely protects the arguments at the basis of the UV self complete quantum gravity.

3 Thermodynamics, area quantization and mass spectrum.

In this section we would like to investigate the thermodynamics of the black hole described by (6) and determine the relation between entropy and area

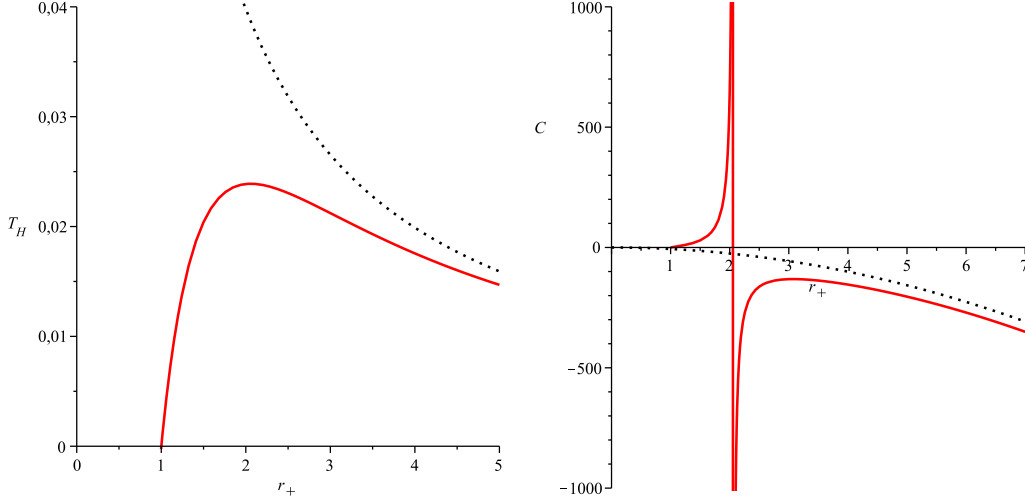


Fig. 2. Solid curves represent the Hawking temperature T_H (left) and the black hole heat capacity C (right) as a function of the horizon radius r_+ in Planck units. Dotted curves represent the corresponding classical results in terms of the Schwarzschild metric.

of the event horizon. It is customary to consider the area law for granted in any case, but this assumption leads to an inconsistency with the Third Law of thermodynamics: extremal black holes have zero temperature but non vanishing area. Here, we stick to the textbook definition of thermodynamical entropy and not to more exotic quantity like Rényi, or entanglement entropy. To cure this flaw, we shall *derive* the relation between entropy and area from the First Law, rather than assuming it.

The Hawking temperature is given by

$$T_H = \frac{1}{4\pi r_+} \left(1 - \frac{2L_P^2}{r_+^2 + L_P^2} \right) \quad (15)$$

while the heat capacity $C \equiv \partial U / \partial T_H$ is

$$C \equiv \frac{\partial M}{\partial r_+} \left(\frac{\partial T_H}{\partial r_+} \right)^{-1} = -2\pi r_+ \left(\frac{r_+^2 - L_P^2}{L_P^2} \right) \frac{(r_+^2 + L_P^2)^2}{r_+^4 - 4L_P^2 r_+^2 - L_P^4}. \quad (16)$$

One can check that for large distances, i.e., $r_+ \gg L_P$ both (15) and (16) coincide with the conventional results of the Schwarzschild metric, i.e., $T_H \approx \frac{1}{4\pi r_+}$ and $C \approx -2\pi r_+^2$ (see Fig. 2). On the other hand at Planckian scales, contrary to the standard result for which a Planckian black hole has a temperature $T_H = M_P/8\pi$, we have that $T_H \rightarrow 0$ as $r_+ \rightarrow r_0 = L_P$ as expected for any extremal configurations. This discrepancy with the classical picture is consistent with the genuine *quantum gravitational* character of the black hole. The Hawking emission is a semi-classical decay where gravity is considered just in terms of a classical spacetime background. Such a semi-classical ap-

proximation breaks down as the Planck scale is approached. This occurs at $r_+ = r_M = \sqrt{2 + \sqrt{5}}L_P \simeq 2.058L_P$ where the temperature admits a maximum corresponding to a pole in the heat capacity. In the final stage of the evaporation, i.e. $L_P < r_+ < r_M$, the heat capacity is positive, the Hawking emission slows down and switches off at $r_+ = L_P$. From a numerical estimate of the maximum temperature one finds $T_H(r_M) = 0.0239M_P$. This implies that the ratio temperature/mass is $T_H/M < T_H(r_M)/M_0 \simeq 0.0239$. As a consequence, no relevant back reaction occurs during all the evaporation process and the metric can consistently describe the system “black hole + radiation” for all $r_+ \geq L_P$.

We can summarize the process with the following scheme:

- “*large*”, far-from-extremality, black holes are semi-classical objects which radiates thermally;
- “*small*”, quasi-extremal, black holes are quantum objects;
- $r = r_M$ is “*critical point*” where the heat capacity diverges. Since $C > 0$ for $r_0 < r_+ < r_M$ and $C < 0$ for $r_M < r_+$, we conclude that a phase transition takes place from large thermodynamically unstable black holes to small stable black holes.

As a matter of fact, the black hole emission preceding the evaporation switching off (often called “SCRAM phase” [11]) might not be thermal. It has been argued that such a quantum regime might be characterized by discrete jumps towards the ground state [19]. To clarify the nature of this mechanism we proceed by studying the black hole entropy profile and the related area quantization.

By integrating the First Law, taking into account that no black hole can have a radius smaller than $r_0 = L_P$, i.e.,

$$S(r_+) = \int_{r_0}^{r_+} \frac{dM}{T_H}, \quad (17)$$

one obtains

$$S(r_+) = \frac{\pi}{L_P^2} (r_+^2 - L_P^2) + 2\pi \ln \left(\frac{r_+}{L_P} \right). \quad (18)$$

We can cast the entropy in terms of the area of the event horizon $\mathcal{A}_+ \equiv 4\pi r_+^2$ as

$$S(\mathcal{A}_+) = \frac{\pi}{\mathcal{A}_0} (\mathcal{A}_+ - \mathcal{A}_0) + \pi \ln (\mathcal{A}_+/\mathcal{A}_0) \quad (19)$$

where $\mathcal{A}_0 = 4\pi L_P^2$ is the area of the extremal event horizon. We remark that the modifications to the Schwarzschild metric, encoded in our model, are in agreement with all the major approaches to quantum gravity, which universally foresee a logarithmic term as a correction to the classical area law [20].

We can check this by performing the limit $r_+ \gg L_P$ for (19) to obtain

$$S(\mathcal{A}_+) \approx \frac{\mathcal{A}_+}{4L_P^2} + \pi \ln \left(\frac{\mathcal{A}_+}{4\pi L_P^2} \right). \quad (20)$$

Conversely for $r_+ \rightarrow L_P$ the entropy vanishes, i.e.,

$$S(\mathcal{A}_+) \approx \frac{4\pi}{L_P} (r_+ - L_P) + O((r_+ - L_P)^2). \quad (21)$$

This result is consistent both with the Third Law of thermodynamics and the entropy statistical meaning. The Planck size, zero temperature, black hole configuration is the unique ground state for holographic screens. Thus, it is a zero entropy state as there is only one way to realize this configuration. To see this we promote the extremal configuration area to the fundamental quantum of area.

$$\mathcal{A}_+ \equiv \mathcal{A}_{n-1} = n \mathcal{A}_0 = 4\pi n L_P^2, \quad (22)$$

where L_P^2 represents the basic information pixel and $n = 1, 2, 3 \dots$ is the number of bytes.³ From the above condition one obtains

$$r_{n-1} \equiv n^{1/2} L_P \quad (23)$$

and

$$M_{n-1} \equiv \frac{1}{2} (n^{1/2} + n^{-1/2}) M_P. \quad (24)$$

Consistently the ground state of the system is $r_0 = L_P$ and $M_0 = M_P$, while for $n \gg 1$ one finds a continuous spectrum of values. This can be checked through the following relation

$$\Delta M_n \equiv M_n - M_{n-1} \sim \frac{1}{4} n^{-1/2} M_P. \quad (25)$$

We notice that for $n \leq 4$ we are in the regime of positive heat capacity $C > 0$ and discrete mass spectrum, while for $n > 4$ we approach the semi-classical limit characterized by negative heat capacity $C < 0$ and continuous mass spectrum, i.e., $\Delta M_n/M_n \leq 1/12$. This confirms that at $r_+ = r_M$, the system undergoes a phase transition from a semi-classical regime to a genuine quantum gravity regime. As a conclusion we have that large black holes decay thermally, while small objects decay quantum mechanically, by emitting

³ We borrow here the names of some units of digital information. In the present context, each byte consists of 4π bits. Each bit, represented by L_P^2 is the basic capacity of information of the holographic screen. In the analogy with the theory of information for which a byte represents the minimum amount of bits for encoding a single character of text, here the byte represents the minimum number of basic pixel L_P^2 for encoding the smallest holographic screen.

quanta of energy (for a recent phenomenological analysis of such kind of decay see [21]). The end-point of the decay is a Planck mass, holographic screen.

The quantization of the area of the holographic screen lets us to disclose further features of the informational content of the holographic screen. We have that the surface density can be written as

$$\sigma_h(n) = \frac{1}{2} \left(\frac{1}{n^{1/2}} + \frac{1}{n^{3/2}} \right) \frac{M_P}{4\pi L_P^2} \quad (26)$$

while the entropy reads

$$S(n) = \pi (n + \ln(n) - 1). \quad (27)$$

From the above relations we learn that while the entropy increases with the number n of bytes, the surface density decreases. This confirms that the extremal configuration is nothing but a single byte, zero-entropy, Planckian density holographic screen.

4 Emergent gravity

In the previous sections we have seen how gravity leads to holography. However this logic can be reversed: holography can lead to gravity. Specifically, it has recently been proposed that gravity might be an emergent phenomenon of some yet unknown microscopic statistical theory [22] (for more recent developments see [23]). As a result, by invoking the universality of thermodynamics, Newton's law can be derived as an entropic force, provided that one generalizes the concept of holographic screen to any generic surface where the information about the matter-energy content can be stored [24]. In this regard an event horizon would be nothing but a special kind of holographic screen having maximum entropy.

We proceed by considering two masses, one test mass m and a source mass M , and a closed surface Ω centered around M and lying between the two masses. We also assume m to be at a distance from Ω smaller with respect to its reduced Compton wavelength $1/m$. The key ingredient for emergent gravity is the entropy associated to the screen Ω . We postulate that the change of entropy near the screen is linear with the radial displacement Δx , i.e.

$$\Delta S_\Omega = 2\pi k_B \frac{mc}{\hbar} \Delta x, \quad (28)$$

where for sake of clarity we have temporarily restored all constants. As a second step we consider the energy associated to the surface, i.e., $U_\Omega = Mc^2$. Being Ω a holographic screen we can assume that the information is encoded

on it in terms of a certain number N of bits. As we discussed before, the fundamental element of information is nothing but a pixel of area L_P^2 . The number of bits is therefore $N = \mathcal{A}_\Omega / L_P^2$ where \mathcal{A}_Ω is the area of the holographic screen. In case of thermal equilibrium all bits are equally likely and we can associate a temperature T to the energy U_Ω via $U_\Omega = \frac{1}{2} N k_B T$. As a result the gravitational force can be obtained in terms of variation of the entropy, namely

$$F \Delta x = T \Delta S_\Omega. \quad (29)$$

At the basis of the above reasoning lies the possibility of associating to the screen a holographic entropy like in the case of an event horizon. To clarify this point we can re-write (28) as [25]

$$\Delta S_\Omega = 8\pi L_P^2 \left(\frac{\partial S_\Omega}{\partial \mathcal{A}_\Omega} \right) \frac{mc}{\hbar} \Delta x. \quad (30)$$

The term $\partial S_\Omega / \partial \mathcal{A}_\Omega$ is in general not known for a generic holographic screen, but, assuming the universality of thermodynamics, it is chosen to be of the same form as for a black hole event horizon. The conventional area-entropy relation for the Schwarzschild black hole leads back to (28) and ultimately, from (29), to the conventional profile of Newton's law. On the other hand for black hole in (6) this is not the case. By using (17) one can prove that the term reads

$$\frac{\partial S_\Omega}{\partial \mathcal{A}_\Omega} = \frac{k_B}{4L_P^2} \left(\frac{r^2 + L_P^2}{r^2} \right). \quad (31)$$

As a result, from (29), the new profile of Newton's law reads

$$F = \left(\frac{c^3 L_P^2}{\hbar} \right) \frac{Mm}{r^2} \left(1 + \frac{L_P^2}{r^2} \right). \quad (32)$$

We notice that the gravitational coupling can now be defined in terms of information bits, i.e., $G = c^3 L_P^2 / \hbar$. At large scales, $r \gg L_P^2$, (32) leads to the conventional Newton's law. On the other hand (32) consistently works until the Planck scale, the ultimate scale at which holographic screens are physically meaningful. Here we have

$$F \approx 2G \frac{Mm}{L_P^2} \quad \text{as} \quad r \simeq L_P. \quad (33)$$

Not surprisingly the major deviations occur in correspondence of the maximum surface density for the holographic screen.

5 Discussion and conclusions

In this paper we have presented two self-regular black hole geometries exhibiting extremal configurations whose mass and radius coincide with the Planck units. We showed that the horizon of the degenerate black hole represents the minimal holographic screen, within which we cannot access to any information about the matter-energy content of the spacetime. On the ground of this line of reasoning, we have chosen to proceed with a detailed analysis of the more compact metric (6), irrespective of its behavior at scales $r < L_P$. We showed that a generic holographic screen is described in terms of the outer horizon of the metric (6), while the inner horizon lies within the prohibited region, i.e., inside the minimal holographic screen. The whole scheme fits into the gravity self-completeness scenario. For sub-Planckian energy scales one has just a quantum particle able to probe at the most distances of the order of its Compton wavelength. By increasing the degree of compression of the particle, one traverses the Planck scale where a collapse into a black hole occurs, before probing a semi-classical regime at trans-Planckian energies. The virtual curvature singularity of the geometry in $r = 0$ is therefore wiped out since in such a context sub-Planckian lengths have no physical meaning. From this vantage point spacetime stops to exist beyond the Planck scale as there is no physical way to access this regime. Thus, the curvature singularity problem is ultimately resolved by giving up the very concept of spacetime at sub-Planckian length scales.

The study of the associated thermodynamic quantities confirmed that at trans-Planckian energies black holes radiate thermally before undergoing a phase transition to smaller, quantum black holes. The latter decay by emitting a discrete spectrum of quanta of energy and reach the ground state of the evaporation corresponding to the minimal holographic screen. We came to this conclusion by quantizing the black hole horizon area in terms of the minimal holographic screen which actually plays the role of a basic information byte. We showed that in the thermodynamic limit, the area law for the black hole entropy acquires a logarithmic correction in agreement with all the major quantum gravity formulations. Along the lines of our reasoning based on the concept of holographic screen we have also addressed the issue of Newton's law emerging as an entropic force. Specifically we have computed the corrections to the standard $1/r$ profile, due to the form of the entropy we have computed for the black hole horizon.

In conclusion, we stress that the line element (6) not only captures the basic features of more “sophisticated” models of quantum gravity improved black holes (e.g. noncommutative geometry inspired black holes [11], loop quantum gravity black holes [26], asymptotically safe gravity black holes [27]), but overcomes some of their current weak points: specifically there is no longer any concern for potential Cauchy instabilities or for conflicts between the gravity self-completeness and the Planck scale spontaneous dimensional reduction

mechanism, as well as, the scenario of the terminal phase of the evaporation for static, non-rotating, neutral black holes. In addition, for its compact form the new metric allows straightforward analytic calculations and opens the route to testable predictions.

A Appendix

For the reader who feels more comfortable in the “safe” framework of general relativity, we add in this appendix some notes about how to derive the metric (6) from Einstein equations in the presence of an effective source term.

It is useful to give the metric a Schwarzschild-like appearance by introducing the *cumulative mass distribution* $m(r) = M r^2 / (r^2 + L_P^2)$

$$ds^2 = - \left(1 - \frac{2m(r)G}{r} \right) dt^2 + \left(1 - \frac{2m(r)G}{r} \right)^{-1} dr^2 + r^2 d\Omega^2, \quad (\text{A.1})$$

Using textbook formulas, one finds

$$m(r) = 4\pi \int_r^\infty dr' (r')^2 T_0^0, \quad r \geq L_P \quad (\text{A.2})$$

where $T_0^0 = -\rho(r)$ is the energy density of the system (see Fig. A.1)

$$\rho(r) = \frac{M}{2\pi r} \frac{L_P^2}{(r^2 + L_P^2)^2}. \quad (\text{A.3})$$

The above energy density quickly vanishes at large distance, i.e. $\rho(r) \rightarrow 0$ at $r \gg L_P$, matching the “vacuum” Schwarzschild metric. Conversely, $\rho \simeq M/8\pi L_P^3$ at short scales, i.e., $r \simeq L_P$. In case of extremal configuration, i.e. $M = M_P$, the density $\rho \simeq 0.0398 M_P / L_P^3$ is less than the Planck density. Thus, quantum gravity effects are negligible and it is safe to use the formalism of general relativity to describe even this extremal case.

By means of the continuity equation $\nabla_\mu T^{\mu\nu} = 0$ one can determine the remaining components of the stress tensor, which turns to be out of the form $T_\mu^\nu = \text{diag}(-\rho, p_r, p_\perp, p_\perp)$. The condition $g_{00} = -g_{11}^{-1}$ determines the equation of state, namely the relation between the energy density and the radial pressure, $p_r = -\rho$. The angular pressure is specified by the conservation of the stress tensor and reads $p_\perp = p_r + \frac{r}{2} \partial_r p_r$. One can check that no violations of energy conditions arise for $r \geq L_P$, a fact that makes the source compatible with the tenets of general relativity in the region where the geometry is properly defined. On the other hand, in the forbidden region $r \leq L_P$, a violation of the strong energy condition discloses the non-classical character of the geometry and quantum gravity effects become unavoidable (see Fig. A.2).

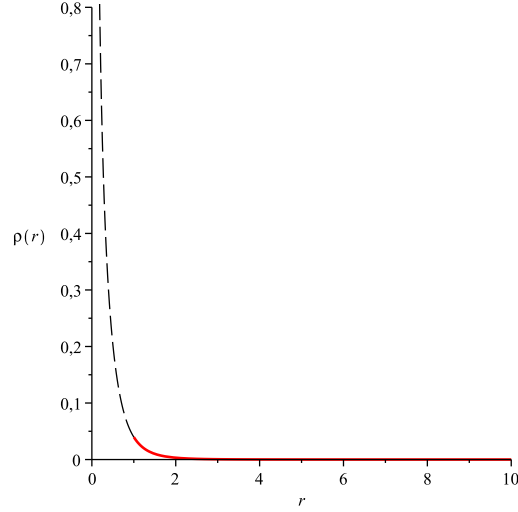


Fig. A.1. The effective energy density (solid curve) $\rho(r)$ in Planck units. The dashed curve represents the energy density which can never be probed since it is behind the ultimate holographic screen.

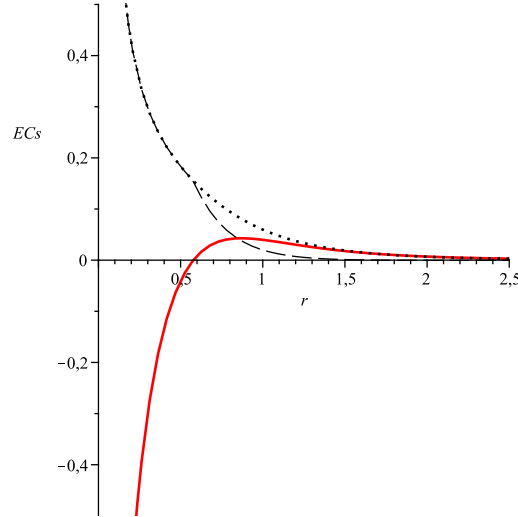


Fig. A.2. Energy conditions in Planck units. The solid curve is the function $(\rho + p_r + 2p_\perp)$ (strong energy condition), the dotted curve is the function $(\rho - |p_\perp|)$ (dominant energy condition) and the dashed curve is $(\rho + p_\perp)$ (null energy condition).

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